

Cavity enhanced light scattering in optical lattices to probe atomic quantum statistics

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Different quantum states of atoms in optical lattices can be nondestructively monitored through off-resonant light scattering into a cavity mode. Angle resolved measurements of photon number and variance provide information about atom-number fluctuations and pair correlations even without access to a single site. Scattering into a standing-wave mode shows structure at angles where classical diffraction gives zero. In particular for transverse illumination no photons are scattered into a cavity for a Mott insulator, while the photon number is proportional to the atom number for a superfluid.

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Ever since the generation of Bose-Einstein condensates (BEC) one has studied the quantum properties of such degenerate gases. A great deal of observations could be explained by the Gross-Pitaevskii equation giving the average atomic density. However, such a mean-field description proved inappropriate for atoms in optical lattices, where one has phase transitions between states of similar density but radically different quantum fluctuations.

Theoretical and experimental studies of optical lattices are of special importance, since they strongly link atomic and condensed matter physics and provide a basis for the implementation of quantum information processing.

Standard methods to characterize the quantum properties of degenerate gases are based on matter-wave interference between atoms released from a trap in time-of-flight measurements [1], which of course destroys the system. For example, “Bragg spectroscopy” uses stimulated matter-wave scattering by laser pulses applied to homogeneous BECs [2] and lattices [3]. Though the light and matter waves can be entangled and both carry information about the atomic quantum state [4], the laser fields are simply used as a tool to stimulate matter waves.

Alternative less destructive methods, based on measurements of light fields only, were proposed mainly for homogeneous BECs [5, 6], but not experimentally implemented yet. Here we show that this is of even greater significance for optical lattices, where various quantum phases show qualitatively distinct light scattering. This can be extremely useful for studying quantum phase transitions in lattices, e.g., between Mott insulator (MI) and superfluid (SF) states, without destruction and allowing subsequent measurements on the very same sample.

Model. We consider N two-level atoms in an optical lattice with M sites. A region of $K \leq M$ sites is illuminated by probe light which is scattered into another mode (cf. Fig. 1). Although, each mode could be a freely propagating field, we will consider cavity modes whose axes can be varied with respect to the lattice. Equivalently, instead of angles, the wavelength of the modes or lattice could be varied. A manybody Hamiltonian for our system is given by

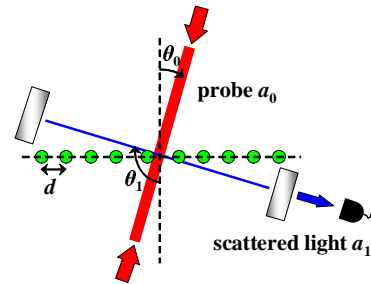


FIG. 1: (Color online) Setup. A lattice is illuminated by a probe at the angle θ_0 which is scattered into a cavity at θ_1 .

$$H = \sum_{l=0,1} \hbar \omega_l a_l^\dagger a_l + \int d^3 \mathbf{r} \Psi^\dagger(\mathbf{r}) H_{a1} \Psi(\mathbf{r}), \quad (1)$$

$$H_{a1} = \frac{\mathbf{p}^2}{2m} + V_{cl}(\mathbf{r}) + \hbar g_0^2 \sum_{l,m=0,1} \frac{u_l^*(\mathbf{r}) u_m(\mathbf{r}) a_l^\dagger a_m}{\Delta_{ma}},$$

where a_0 (a_1) are the annihilation operators of the probe (scattered) light with the frequencies $\omega_{0,1}$, wave vectors $\mathbf{k}_{0,1}$, and mode functions $u_{0,1}(\mathbf{r})$; $\Psi(\mathbf{r})$ is the field operator for atoms. In the effective single-atom Hamiltonian H_{a1} , \mathbf{p} and \mathbf{r} are the momentum and position operators of an atom, m is its mass, and g_0 is the atom-light coupling constant. We consider off-resonant scattering where the detunings between fields and atomic transition $\Delta_{la} = \omega_l - \omega_a$ are much larger than the spontaneous emission rate and Rabi frequencies. Thus, in H_{a1} the adiabatic elimination of the upper state, assuming linear dipoles with adiabatically following polarization, was used. In principle, the classical trapping potential $V_{cl}(\mathbf{r})$, here independent of $a_{0,1}$, can also play a role of the probe.

Assuming weak fields $a_{0,1}$, we expand $\Psi(\mathbf{r})$ in Eq. (1) using localized Wannier functions corresponding to $V_{cl}(\mathbf{r})$ and keep only the lowest vibrational state at each site: $\Psi(\mathbf{r}) = \sum_{k=1}^M b_k w(\mathbf{r} - \mathbf{r}_k)$, where b_k is the annihilation operator of an atom at site k with coordinate \mathbf{r}_k . Substituting this expansion in Eq. (1), one can get a general-

ized Bose-Hubbard Hamiltonian including light scattering. However, in contrast to previous work [7] and “Bragg spectroscopy” [3], we do not consider lattice excitations here and focus on scattering from atoms in a prescribed state, which is not necessarily the ground state.

Neglecting atomic tunneling, the Hamiltonian reads:

$$H = \sum_{l=0,1} \hbar \omega_l a_l^\dagger a_l + \hbar g_0^2 \sum_{l,m=0,1} \frac{a_l^\dagger a_m}{\Delta_{ma}} \left(\sum_{i=1}^K J_{i,i}^{lm} \hat{n}_i \right),$$

where $\hat{n}_i = b_i^\dagger b_i$. If $K < M$, the number of radiating atoms (determined by $\hat{N}_K = \sum_{i=1}^K \hat{n}_i$) may fluctuate, in contrast to the conserved total atom number given by $\hat{N} = \sum_{i=1}^M \hat{n}_i$. For a deep lattice the coefficients $J_{i,i}^{lm} = \int d\mathbf{r} w^2(\mathbf{r} - \mathbf{r}_i) u_l^*(\mathbf{r}) u_m(\mathbf{r})$ can be approximated as $J_{i,i}^{lm} = u_l^*(\mathbf{r}_i) u_m(\mathbf{r}_i)$ neglecting details of the atomic localization.

The Heisenberg equation for the scattered light in the frame rotating with ω_0 ($\Delta_{01} = \omega_0 - \omega_1$) thus reads:

$$\begin{aligned} \dot{a}_1 = i \left[\Delta_{01} - \frac{g_0^2}{\Delta_{1a}} \sum_{i=1}^K |u_1(\mathbf{r}_i)|^2 \hat{n}_i \right] a_1 \\ - i \frac{g_0^2 a_0}{\Delta_{0a}} \sum_{i=1}^K u_1^*(\mathbf{r}_i) u_0(\mathbf{r}_i) \hat{n}_i - \kappa a_1, \end{aligned} \quad (2)$$

where κ is the cavity decay rate and a_0 will be assumed a classical field given by a c-number constant.

Light properties. Though the dispersion shift of a cavity mode is sensitive to atom statistics through \hat{n}_i , we assume it is much smaller than κ or Δ_{01} . A stationary solution of Eq. (2) for the field amplitude and photon number n_{ph} then reads

$$a_1 = C \hat{D}, \quad n_{ph} = a_1^\dagger a_1 = |C|^2 \hat{D}^* \hat{D}, \quad \hat{D} = \sum_{i=1}^K A_i \hat{n}_i, \quad (3)$$

with $C \equiv i g_0^2 a_0 / [\Delta_{0a}(i\Delta_{01} - \kappa)]$ and the geometry dependent coefficients $A_i(\theta_0, \theta_1) \equiv u_1^*(\mathbf{r}_i) u_0(\mathbf{r}_i)$.

This expression of the light operators in terms of the atomic ones in Eq. (3) is a central result here, which we will now use to study the properties of the scattered field.

Let us consider a 1D lattice with period d and atoms trapped at $x_m = md$ ($m = 1, 2, \dots$), which gives the mode functions $u_{0,1}(\mathbf{r}_m) = \exp(imk_{0,1}d)$ for traveling and $u_{0,1}(\mathbf{r}_m) = \cos(mk_{0,1}d)$ for standing waves with $k_{0,1x} = |\mathbf{k}_{0,1}| \sin \theta_{0,1}$ (cf. Fig. 1). For the atomic quantum state we use the assumptions: (i) the mean atom number at all sites is $\langle \hat{n}_i \rangle = n = N/M$ ($\langle \hat{N}_K \rangle = N_K \equiv nK$) and (ii) the pair correlations are $\langle \hat{n}_i \hat{n}_j \rangle = \langle \hat{n}_a \hat{n}_b \rangle$ for any $a \neq b$, which is valid for a deep lattice.

Thus we have $\langle a_1 \rangle \sim \langle \hat{D} \rangle = \sum_{i=1}^K A_i \langle \hat{n}_i \rangle = nA$ showing that the field amplitude only depends on the mean density and exhibits the angular distribution of classical

diffraction $A(\theta_0, \theta_1) \equiv \sum_{i=1}^K A_i(\theta_0, \theta_1)$ displaying diffraction maxima and minima. However, the central point now is that the photon number (3) is not simply the amplitude squared, but we get

$$\langle \hat{D}^* \hat{D} \rangle = \langle \hat{n}_a \hat{n}_b \rangle |A|^2 + (\langle \hat{n}^2 \rangle - \langle \hat{n}_a \hat{n}_b \rangle) \sum_{i=1}^K |A_i|^2, \quad (4a)$$

$$\begin{aligned} R(\theta_0, \theta_1) \equiv \langle \hat{D}^* \hat{D} \rangle - |\langle \hat{D} \rangle|^2 = \\ = \langle \delta \hat{n}_a \delta \hat{n}_b \rangle |A|^2 + (\langle \delta \hat{n}^2 \rangle - \langle \delta \hat{n}_a \delta \hat{n}_b \rangle) \sum_{i=1}^K |A_i|^2, \end{aligned} \quad (4b)$$

where $\delta \hat{n}_i = \hat{n}_i - n$, which depends on the density-density correlations $\langle \hat{n}_i \hat{n}_j \rangle$ different for particular states. Thus, the intensity is sensitive to atomic quantum statistics. Besides the classical angle dependence $|A|^2$, the second term in Eq. (4a) reflects fluctuations and has a completely different dependence. Equation (4a) is similar to that of Refs. [5] for homogeneous BECs, where the intensity has two terms: a “coherent,” density dependent part, and “incoherent,” fluctuation dependent part. However, for a lattice, this would be only valid for zero correlations $\langle \hat{n}_a \hat{n}_b \rangle = n^2$, which in general is not true and leads to the observable difference between states with and without nonlocal correlations. Moreover, Eq. (4b) for the noise quantity R shows that the classical distribution $|A|^2$ appears only for nonzero pair correlations, since, in a lattice, scattering is sensitive not only to the periodic density, but also to periodic fluctuations. Note that analysis of the quadrature variances gives results similar to R .

For two traveling waves, Eq. (4a) gives the structure factor considered in Ref. [6] on homogeneous BECs. We show that a more general form including standing waves gives new measurable quantities beyond structure factor.

The intensity fluctuations of the scattered light depend on the fourth moments of the atomic number operators and four-point density correlations $\langle \hat{n}_i \hat{n}_j \hat{n}_k \hat{n}_l \rangle$. For example the photon-number variance is given by $(\Delta n_{ph})^2 = \langle n_{ph}^2 \rangle - \langle n_{ph} \rangle^2 = |C|^4 (\langle |\hat{D}|^4 \rangle - \langle |\hat{D}|^2 \rangle^2) + |C|^2 \langle |\hat{D}|^2 \rangle$.

To discuss specific examples of different scattering properties we summarize statistical properties of typical states in Table I. From the light-scattering point of view, the most classical atomic state corresponding to pointlike atoms is a MI. Here the atom number at each site \hat{n}_i does not fluctuate and we have no pair correlations. Hence we see from Eq. (4a) that the zeros of classical diffraction [$A(\theta_0, \theta_1) = 0$] are exact zeros of light intensity.

This is different for a SF where each atom is delocalized over all sites leading to number fluctuations at a given site and at $K < M$ sites; the atoms at different sites are anticorrelated. At a classical diffraction zero we still find a photon number proportional to the atom number N .

A coherent state approximates a SF with fluctuations at any sites but zero pair correlations. In the limit $N, M \rightarrow \infty$, but finite n , it well describes scattering from

	MI	SF	Coherent
$ \Psi\rangle$	$\prod_{i=1}^M n_i\rangle_i$	$\frac{1}{\sqrt{M^N N!}} (\sum_{i=1}^M b_i^\dagger)^N 0\rangle$	$e^{-\frac{N}{2}} \prod_{i=1}^M e^{\sqrt{\frac{N}{M}} b_i^\dagger} 0\rangle_i$
$\langle \hat{n}_i^2 \rangle$	n^2	$n^2(1 - 1/N) + n$	$n^2 + n$
$\langle \Delta n_i \rangle^2$	0	$n(1 - 1/M)$	n
$\langle \hat{N}_K^2 \rangle$	N_K^2	$N_K^2(1 - 1/N) + N_K$	$N_K^2 + N_K$
$\langle \Delta N_K \rangle^2$	0	$N_K(1 - K/M)$	N_K
$\langle \hat{n}_a \hat{n}_b \rangle$	n^2	$n^2(1 - 1/N)$	n^2
$\langle \delta \hat{n}_a \delta \hat{n}_b \rangle$	0	$-N/M^2$	0

TABLE I: Statistical quantities of typical atomic states.

a small region ($K \ll M$) of a partially illuminated superfluid (SF_K). However, we proved that even in this limit it fails to describe scattering at angles of Bragg maxima from a large lattice region ($K \sim M$).

Example. Let us now show the most striking predictions of this model at the basic example of a probe transverse to the lattice ($\theta_0 = 0$, cf. Fig. 1). The scattered light is collected in a cavity along the lattice ($\theta_1 = \pi/2$) with atoms trapped at the antinodes ($d = \lambda/2$) [7, 8].

The operator $\hat{D} = \sum_{k=1}^K (-1)^{k+1} \hat{n}_k$ (3) here gives zero average field amplitude independently on the atomic state (the difference between large odd and even K is negligible). This reflects the opposite phase of light scattered from atoms separated by $\lambda/2$ (diffraction minimum). However, the cavity photon-number is proportional to $\langle \hat{D}^* \hat{D} \rangle = (\langle \hat{n}^2 \rangle - \langle \hat{n}_a \hat{n}_b \rangle) K$ [cf. Eq. (4a)], which is determined by statistics of a particular state. Thus, atoms in a MI state scatter no photons, while a SF scatters number of photons proportional to the atom number:

$$\begin{aligned} \langle a_1 \rangle_{\text{MI}} &= \langle a_1 \rangle_{\text{SF}} = 0, \quad \text{but} \\ \langle a_1^\dagger a_1 \rangle_{\text{MI}} &= 0, \quad \langle a_1^\dagger a_1 \rangle_{\text{SF}} = |C|^2 N_K. \end{aligned}$$

Hence, already the mean photon number provides information about quantum statistics of atoms.

Moreover, the photon number fluctuations $(\Delta n_{\text{ph}})^2$ are also different for various atomic states. In the MI state, the variance $(\Delta |D|^2)_{\text{MI}}^2 = \langle \hat{D}^4 \rangle_{\text{MI}} - \langle \hat{D}^2 \rangle_{\text{MI}}^2 = 0$, whereas in SF, there is a strong noise $(\Delta |D|^2)_{\text{SF}}^2 \approx 2N_K^2$.

Let us remark that the coupled light-matter dynamics in a cavity can lead to a new self-organized phase [8] with atoms trapped at every second site ($d = \lambda$). This would be immediately noticed here, as we have $\hat{D} = \sum_{k=1}^K \hat{n}_k = \hat{N}_K$ (3). If this state is a MI with $d = \lambda$, the number of photons $\langle a_1^\dagger a_1 \rangle_{\text{Self-org}} = |C|^2 N_K^2$ is proportional to the atom number squared and has a superradiant character.

Angular distributions. We will quantitatively discuss angular intensity distributions for scattering between two traveling waves, where Eq. (4b) reduces to

$$R = \langle \delta \hat{n}_a \delta \hat{n}_b \rangle \frac{\sin^2(K\alpha_-/2)}{\sin^2(\alpha_-/2)} + (\langle \hat{n}^2 \rangle - \langle \delta \hat{n}_a \delta \hat{n}_b \rangle) K. \quad (5)$$

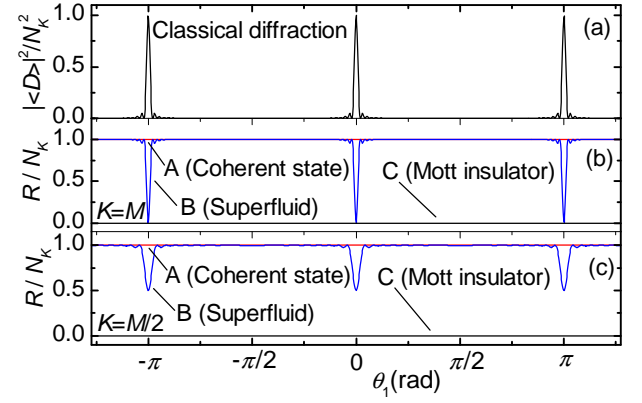


FIG. 2: (Color online) Intensity angular distributions for two traveling waves. (a) Intensity of classical diffraction; (b) noise quantity R (5) for coherent atomic state (constant 1, line A), SF with all sites illuminated $K = M$ (curve B), and MI state (constant 0, line C); (c) the same as in (b) but for partially illuminated SF with $K = M/2$. $N = M = 30$, $\theta_0 = 0$.

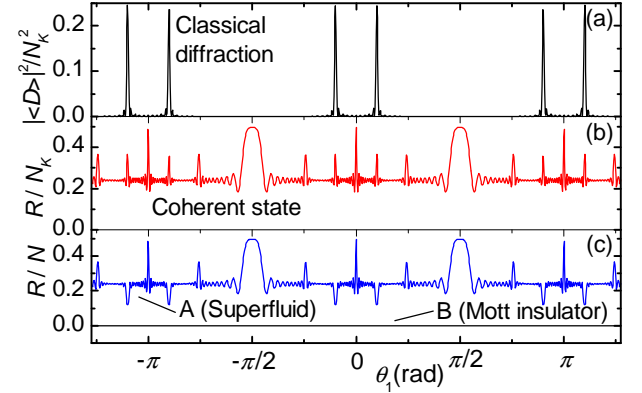


FIG. 3: (Color online) Intensity angular distributions for two standing-wave modes. (a) Intensity of classical diffraction; (b) noise quantity for coherent state; and (c) for SF (curve A) and MI (constant 0, line B). $N = M = K = 30$, $\theta_0 = 0.1\pi$.

While the first term with $|A|^2$ reproduces the angle dependence of classical diffraction with $\alpha_- = k_{0x}d \sin \theta_0 - k_{1x}d \sin \theta_1$, the second term in Eq. (4b) is simply isotropic. Thus, the noise quantity is zero for MI, $R_{\text{MI}} = 0$, nonzero but isotropic for the coherent state, $R_{\text{Coh}} = N_K$, and angle dependent for a SF, $R_{\text{SF}_K} = -N/M^2 \sin^2(K\alpha_-/2)/\sin^2(\alpha_-/2) + N_K$. Note that for a SF even small pair correlations $\langle \delta \hat{n}_a \delta \hat{n}_b \rangle = -N/M^2$ give a large contribution near diffraction maxima ($\alpha_- = 2\pi l$, $l = 0, 1, \dots$), where the geometric factor is K^2 , invalidating the coherent-state approximation.

Figure 2 displays these angular distributions. The classical diffraction $|D|^2$ with the only possible zero-order maxima at $\theta_1 = 0, \pi$ ($d = \lambda_{0,1}/2$, $\theta_0 = 0$) is shown in Fig. 2(a). The noise quantity R for the coherent (constant lines A) and SF_K (curves B) states are plotted in Figs. 2(b) and 2(c). For MI, $R = 0$ is displayed by lines

C. In SF, there is a noise suppression at maxima, which is total for all sites illuminated, $K = M$ [cf. Fig. 2(b)], and partial for $K = M/2$ [cf. Fig. 2(c)].

In a maximum, \hat{D} (3), is reduced to \hat{N}_K . Thus, the field amplitude is determined by $N_K = nK$, the intensity depends on $\langle \hat{D}^* \hat{D} \rangle = \langle \hat{N}_K^2 \rangle$, while $R = (\Delta N_K)^2$ gives the atom number variance at K sites, which reflects the total and partial noise suppression in Figs. 2(b) and 2(c), since $\langle N_K \rangle$ fluctuates for $K < M$. In diffraction “minima” (for $K \gg 1$ this is valid everywhere outside maxima), the field is zero, but the intensity is proportional to $\langle \hat{n}^2 \rangle - \langle \hat{n}_a \hat{n}_b \rangle$. Under scattering of spatially incoherent light, the intensity is isotropic and proportional to $\langle \hat{n}^2 \rangle$.

So, in optical experiments, varying the geometry (angles or wavelengths), the global statistics of $K \leq M$ sites, local single-site statistics, and pair correlations can be obtained even without a single-site access. Thus, light scattering gives a way to distinguish between atomic states. As shown by Eq. (5) and Fig. 2, MI and SF_M states are different in diffraction “minima” and in incoherent light. They are indistinguishable (for traveling waves) in maxima, because of the atom number conservation. The SF_M and coherent states differ in maxima only. The MI and coherent state are different at any angles.

The noise quantity or photon statistics are different in orders of N_K for various states. Nevertheless, for large N_K , there could be practical problems to subtract large values in a maximum. In some of Refs. [5], this even led to a conclusion about the state indistinguishability by intensity measurements. In contrast to homogeneous BECs, in lattices, this problem has a natural solution: measurements outside maxima are free of the strong classical-like part of scattering and thus directly reflect fluctuations.

A classical analogy of different light scattering consists in different density fluctuations. A quantum treatment gives a deeper insight. Superfluid state is a superposition of all possible multisite Fock states giving distributions of N atoms at M sites. Various Fock states become entangled to scattered light of different phases and amplitudes. In contrast to a classical case (and MI with the only Fock state), light fields entangled to various distributions do not interfere with each other (due to the orthogonality of the Fock states), which, e.g., explains the zero amplitude but nonzero photon number in a diffraction minimum.

If at least one of the modes is a standing wave, the angle dependences become much richer. Besides new classical maxima given by $\alpha_{\pm} = k_{0x}d \sin \theta_0 \pm k_{1x}d \sin \theta_1$, the second, “noise,” term in Eqs. (4a) and (4b) is also not isotropic. It includes a sum of the geometric coefficients squared, which is equivalent to effective doubling of the lattice period (or light frequency doubling) and leads to the appearance of new narrow features at angles, where classical diffraction predicts zero. In Fig. 3, a case of two standing waves is shown. Due to the effective period doubling (determined by $2\alpha_{0,1} = 2k_{0,1x}d \sin \theta_{0,1}$ and $2\alpha_{\pm}$), new features at the angles of, e.g., effective first-order

diffraction maxima appear, though classically only the zero-order maxima are still possible.

The angle dependence of the photon number variance $(\Delta n_{\text{ph}})^2$ determined by $(\Delta |D|^2)^2$ shows anisotropic features due to effective period doubling even in the case of two traveling waves. For the coherent state, the light at a maximum displays strong noise $[(\Delta |D|^2)^2 = 4N_K^3 + 6N_K^2 + N_K]$ because $\langle |\hat{D}|^4 \rangle = N_K^4 + 6N_K^3 + 7N_K^2 + N_K$ and $\langle |\hat{D}|^2 \rangle = N_K^2 + N_K$, stronger than the isotropic component (N_K^2 in highest order of N_K) and new features at $\theta_1 = \pm \pi/2$ (for $\theta_0 = 0$, $2N_K^2$ in highest order of N_K). In SF_M, the noise at maxima can be suppressed, while at other angles it is nearly equal to that of the coherent state. In MI, $(\Delta |D|^2)^2 = 0$ for all angles. Distinguishing between atomic states by light statistics is similar to that by the intensity measurements considered.

In conclusion, we have shown that the quantum state of atoms in optical lattices can be nondestructively monitored by measuring scattered light [9]. In contrast to homogeneous BECs, scattering from lattices exhibits advantageous properties such as suppression of the classical scattering amplitude in Bragg minima, access to local and nonlocal correlations, and angular distributions richer than classical diffraction. In general, various optical phenomena and quantities depending nonlinearly on the atomic number operators, such that their expectation values cannot be found from the average density, should reflect quantum statistics of atoms. For example, the dispersion of a medium, neglected in this work, will provide a spectral method of the quantum state characterization [10].

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